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A Novel Analytical Formulation for Deriving Net Force Directly from Instantaneous Displacement Equations

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صياغة تحليلية مبتكرة لاشتقاق القوة الصافية مباشرة من معادلات الإزاحة اللحظية

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Abstract:

Observations in classical mechanics education reveal that students frequently face challenges when deducing the net force ($\vec{F}_{net}$)from the instantaneous displacement equation ($\vec{r}$(t)). The conventional approach, rooted in Newton's Second Law ($\vec{F}_{net}$= m$\vec{a}$), necessitates a series of sequential differentiations which can be prone to error. To streamline these derivations and provide a more direct analytical tool for dynamic analysis, this paper introduces a derived formula that allows for the straightforward determination of net force directly from displacement. This work facilitates a more efficient connection between kinematic descriptions and their underlying dynamic causes, offering both pedagogical benefits and analytical precision.

Keywords: Classical Mechanics, Instantaneous Displacement, Net Force Derivation, Kinematics-Dynamics Link.

الملخص

لقد لاحظنا اثناء تدريسنا لمقرر الميكانيكا الكلاسيكية أن الطلاب يواجهون تحديات متكررة عند استنتاج محصلة القوى($\vec{F}_{net}$)من معادلة الإزاحة اللحظية ($\vec{r}$(t)) في المنهج التقليدي، القائم على قانون نيوتن الثاني ($\vec{F}_{net}$= m$\vec{a}$)،والذي يتطلب سلسلة من عمليات التفاضل المتتابعة التي قد تكون عرضة للأخطاء الرياضية. ومن أجل تبسيط هذه الاشتقاقات وتقديم أداة تحليلية أكثر مباشرة للتحليل الديناميكي، تقدم هذه الورقة البحثية صيغة رياضية مستنتجة تسمح بالتحديد المباشر لمحصلة القوى انطلاقاً من الإزاحة. يساهم

هذا العمل في تسهيل الربط بين الأوصاف الكينماتيكية ومسبباتها الديناميكية الأساسية بكفاءة عالية، مما يوفر فوائد تعليمية جمة ودقة تحليلية متقدمة.

الكلمات المفتاحية: الميكانيكا الكلاسيكية، الإزاحة اللحظية، اشتقاق القوة المحصلة، اشتقاق القوة المحصلة

1. Introduction

The field of dynamics is central to classical mechanics, primarily focusing on the causal link between a particle's motion and the governing forces [1]. This relationship is traditionally quantified by Newton’s Second Law, which defines the net force ($\vec{F}_{net}$)as the product of an object's mass (m) and its acceleration ($\vec{a}$) [2]. While the theoretical framework is well-established, its practical implementation—especially in pedagogical and complex analytical contexts—often involves a redundant multi-stage differential process [3].

In standard practice, determining the resultant force from an instantaneous displacement function ($\vec{r}$(t)) requires a two-step differentiation: first to derive velocity ($\vec{v}$(t)), and subsequently to obtain acceleration. This fragmented derivation can sometimes obscure the direct physical correlation between position and dynamics, leading to analytical inefficiencies in both classroom settings and computational modeling [2, 3].

Recent studies highlight that the mathematical complexity in derivation often acts as a barrier to conceptual understanding in physics [7]. This paper addresses this gap by offering a streamlined analytical route.

Motivated by the need to streamline this process, the present work introduces a unified analytical approach. We propose a direct formulation that bypasses intermediate kinematic variables, allowing for the immediate inference of force from displacement. By establishing this consolidated framework, this paper aims to simplify the computational transition from kinematics to dynamics, providing a more intuitive and efficient tool for both educational and research applications in mechanical systems.

2. Classical Derivation: Mathematical Derivation of the Direct Force Equation

2.1 Establishing the Foundation

The derivation begins by recalling the fundamental principles of Newtonian mechanics. The motion of a particle of constant mass m is entirely described by its instantaneous position vector ($\vec{r}(t)$)

According to Newton's Second Law of Motion:

$$\vec{F}_{net} = m\vec{a} \qquad (Equation 1)$$

2.2 Expressing Acceleration through Displacement

The instantaneous velocity ($\vec{v}(t)$)is defined as the first-time derivative of the position vector, and the instantaneous acceleration ($\vec{a}(t)$) is defined as the first-time derivative of the velocity vector, which is equivalent to the second time derivative of the position vector $\left(\vec{r}(t)\right)$.
Given the instantaneous displacement equation:

$$\vec{r} = \vec{r}(t)$$

The acceleration vector ($\vec{a}(t)$) is:

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2} \qquad (Equation\ 2)$$

2.3The Direct Force Equation

By substituting the expression for acceleration (Equation 2) directly into Newton's Second Law (Equation 1), we obtain the generalized and direct force equation:

$$\vec{F}_{net}(t) = m.\frac{d^2(\vec{r})}{dt^2}$$

This equation provides a single, unified step to determine the resultant force from the displacement function.
Building upon this direct relationship, the following section introduces a generalized power-law formulation that further simplifies force inference for specific motion profiles.

3. Proposed Methodology: A New Mathematical Derivation for Net force

The Proposed General Formula:

Considering a particle of mass m whose displacement s follows the power-law function $s(t) = At^n$, the net force $\sum F$can be expressed as a direct function of time and the power index n:

$$\sum F = mAn(n-1)t^{n-2}$$

Analysis of Physical States based on n:

- **Case 1 (n=0):** The body is at rest (s = A), leading to $\sum F$= 0.
- **Case 2 (n=1):** The body moves with a constant velocity (v = A), and the acceleration is zero, thus $\sum F$= 0.
- **Case 3 (n>1):** The body experiences an increasing acceleration, indicating a time-dependent net force.
- **Case 4 (0<n<1):** The body undergoes deceleration (retarding force).

4. Physical Applications and Significance

The proposed generalized formula $\sum F = mAn(n-1)t^{n-2}$ provides a unified framework to describe various physical phenomena by simply adjusting the power index n. This section explores the practical implications of the formula across different motion regimes:

4.1Motion under Constant Gravity (n = 2)

When n = 2, the displacement equation represents a quadratic relationship with time, $s(t) = At^2$

Substituting n = 2 into the proposed formula yields:

$$\sum F = mA(2)(2-1)t^{2-2} = 2mA$$

Significance: The net force is constant and independent of time. This perfectly aligns with the classical physics of Free Fall, where 2A represents the gravitational acceleration (g), and the net force is the object's weight (W = mg).

4.2Constant Jerk Systems (n = 3)

In engineering applications where acceleration changes at a constant rate (known as "Jerk"), the displacement follows a cubic function $s(t) = At^3$.

Substituting n = 3 yields:

$$\sum F = 6mAt$$

Significance: This describes systems where the required force must increase linearly with time, such as in the launch phase of high-speed elevators or advanced propulsion systems designed to minimize mechanical stress.

4.3Sub-linear Dynamics and Deceleration (0 < n < 1)

For indices between 0 and 1, the formula describes motion where the velocity decreases over time.

Significance: This is highly applicable in Fluid Dynamics and the study of particles moving through high-viscosity mediums. The formula provides a direct method to calculate the "Retarding Force" without complex differential modeling.

Table 1: Summary Table for the applications

Motion Regime	Power index (n)	Resulting Net force $\sum F$	Physical example
Static Equilibrium	n=0	0	Object at rest
Uniform Motion	n=1	0	Constant velocity
Uniform Acceleration	n=2	2mA (constant)	Free fall
Variable Acceleration	n=3	6mAt (Linear)	Constant Jerk Motion

5. Comparative Analysis of Analytical Efficiency

To demonstrate the practical advantage of the proposed general equation, a comparison is conducted between the traditional Newtonian derivation and the proposed direct method.

5.1The Traditional Procedural Burden

In the classical approach, the transition from a displacement function $s(t) = At^n$ to the net force $\sum F$ requires a two-step differential operation:

1. **First Differentiation: Calculating instantaneous velocity,**

$$v(t) = \frac{ds}{dt} = nAt^{n-1}$$

2. **Second Differentiation: Calculating instantaneous acceleration,**

$$a(t) = \frac{dv}{dt} = n(n-1)At^{n-2}$$

3. **Application of Mass: Multiplying by mass to find $\sum \boldsymbol{F}$=ma(t).**

This sequential process increases the likelihood of algebraic errors, especially in complex or high-order power functions.

5.2 The Proposed Direct Pathway

The proposed formula $\sum F = mAn(n-1)t^{n-2}$ bypasses these intermediate stages. It allows for a single-step mapping from the displacement parameters (A, n) directly to the dynamic resultant force.

Table 2: Efficiency Comparison between Methods

Feature	Traditional Newtonian Method	Proposed Direct Method
Operational Steps	Three distinct stages (v, a, F)	Single-step operation
Mathematical Tool	Double differentiation	Direct coefficient substitution
Error Probability	Moderate (due to sequential steps)	Minimal (direct calculation)
Computational Time	Higher (requires intermediate variables)	Optimized (direct output)
Pedagogical Clarity	Fragmented (velocity vs. acceleration)	Unified (Displacement to Force)

6. Conclusion

This study has successfully introduced a direct analytical formulation for deriving net force from instantaneous displacement equations, effectively bypassing the traditional, multi-step differentiation process. By establishing the general equation $\sum F = mA_n(n-1)t^{n-2}$, we have provided a tool that enhances both computational speed and analytical accuracy across various physical states.

As demonstrated in our comparative analysis, this approach significantly reduces the procedural burden and minimizes potential algebraic errors. Integrating kinematics and dynamics remains a central challenge in physics pedagogy, necessitating more unified mathematical tools like the one proposed in this study [8].

This consolidated framework not only simplifies the transition between kinematic descriptions and dynamic causes but also offers a more intuitive pathway for students and researchers alike. Future work could explore the integration of this direct method into automated physical simulation software to further optimize dynamic modeling.

Compliance with ethical standards

Disclosure of conflict of interest

The authors declare that they have no conflict of interest.

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